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PHYSICAL CONSIDERATIONS CONCERNING THE DESIGN OF THE BEVATRON

by

Lloyd Smith

University of California
Radiation Laboratory

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The plans now being developed for magnetic resonance accelerators, here called the bevatron which are to produce protons of energies up to 10 Bev represent the latest stage in the quest for high energy particles. The feasibility of obtaining such powerful tools for investigating the fundamental properties of matter has been established since the end of the war, thanks to the discovery of the so-called synchrotron principle and its successful application to smaller machines. In this article we shall first discuss the principles from which the bevatron is to be constructed and trace a cycle of operation of an ideal machine in some detail, and then go into the question of whether a real machine can be constructed which will approximate the ideal case closely enough.

PRINCIPLES OF OPERATION

To understand the operation of the bevatron, let us first consider the basic requirements of an ultra-high energy accelerator. There are, in a broad sense, two of these. First, there must be a means of supplying energy to the particles in many small steps, since the voltages available in the laboratory are much smaller than the final energies sought. Second, there must be a means of guiding the particles surely and precisely over the tremendously long path they must follow while acquiring their energy. Requirement two, which is often not mentioned in elementary explanations of the cyclotron, is of equal importance with the first, for unless the particles are confined to definite paths on a controllable time schedule, any small disturbance will cause them to get out of step or throw them into the walls of the accelerating chamber. If this happens, a machine designed for 100 Bev will give no more high energy particles than a flashlight battery, for none will reach the end of the trip.

Requirement one is fulfilled by placing the accelerating system in a magnetic field so that the same accelerating electrode may be used over and over again. Requirement two needs more careful consideration.

A particle in the accelerating system has in general three dimensions in which to move, and we must be sure that it is constrained in all three. More precisely, we must be able to make it stay away from obstructions in the chamber at all times and to make it arrive at the accelerating gap or gaps at just the right times to receive its correct ration of energy.

A magnetic field, when properly shaped, has strong focussing properties, and so it has been the custom in constructing cyclotrons and betatrons to avoid introducing extra devices by using the guiding magnetic field to define a stable "groove" insofar as this is possible. A lateral stability is in fact already present even if the field is uniform. This comes about in the following way. A particle of a certain energy in a magnetic field will follow a circle whose radius is determined by the field and the energy. This circle we shall hereafter call the instantaneous circle. On the instantaneous circle the magnetic force just balances the centrifugal force, so that a particle moving on this circle will stay there unless some extraneous influence throws it off. Now suppose that something has driven it off, say, outwards.

The centrifugal force, which decreases with increasing radius, is now less, but the magnetic force is not, and so there is a net force on the ion back toward the instantaneous circle. The same thing holds in reverse inside the instantaneous circle, and so we say that this circle is laterally stable; a particle on it stays on it, and a particle away from it is pulled toward it. Moreover, the magnetic field does not have to be uniform for the instantaneous circle to be laterally stable — the only condition is that it fall off more slowly than the centrifugal force as the distance from the center of the magnet increases.

To secure vertical stability, the standard trick is to shape the pole pieces of the magnet in such a way that the field decreases somewhat with increasing radius. If this is done, the magnetic lines of force bulge outward toward the weak field region. Because of the bulging, an ion which happens to be above the center plane of the magnet gap will feel, instead of an exactly radial force, a force which has a slight downward component. The opposite is true for an ion below the center plane, while one on the center plane feels only the radial force. Our definition of stability is thereby fulfilled—as long as the field decreases with increasing radius, there will be a vertically stable groove at the center plane of the gap.

Thus, two of the directions in which the ion could wander off are under absolute control at all times if the magnetic field decreases with increasing radius, but does so less rapidly than the centrifugal force, which is proportional to $1/r$.

We cannot appeal so directly to the cyclotron* for guidance in handling the remaining direction, for it is just the lack of stability fore and aft which eliminates the cyclotron from consideration as a billion volt accelerator. But in this instability lies the key to the solution, so let us examine it in a little detail.

The limitation on the cyclotron occurs because of the relativistic increase of the particle's mass with energy. Because of it, the rotation frequency of the particle drops behind the frequency of the accelerating system; the particle does not stay at the "correct" azimuth (that corresponding to peak potential), but falls farther and farther behind, eventually even getting into the negative portion of the radio-frequency cycle, where it is decelerated. Before the war it was hoped to overcome this lag somewhat by using tremendous accelerating potentials, so that the whole process would be over before the particle had a chance to slip too far behind. But what, actually, is the consequence of this frequency lag? While the ion is still at an azimuth† which corresponds to a positive point on the rf wave it will continue to be accelerated, though not as rapidly as before, and so will still spiral outward, the rotation frequency decreasing all the time. When it has fallen back to the negative part of the rf wave it will begin to be decelerated and so start to spiral inward, and, most important, its frequency will start back toward normal. In fact, by the time it reaches an azimuth corresponding to the peak negative potential, it will again have just the right frequency, but unfortunately will have lost all its energy, too, and will be back at the center of the cyclotron, whence it started. But one extra variable introduced into the system is enough to put this process to practical use — suppose that while the ion was out at a fairly large radius the frequency of the oscillator had been lowered until it was almost equal to the rotation frequency of the ion. Then soon after the ion slid into the negative part of the rf cycle it would have the "right" frequency, and as the deceleration continued, it would take on too high a frequency and so would start to catch up to the rf wave again. Thus a sort of oscillation would be set up, with the ion spiralling in and out about a certain average radius and the ion moving ahead of and behind a certain azimuth. If the field and rf frequency are not changed any more, this spiralling will continue indefinitely with no net energy gain or loss. The average circle, determined solely by the field and the rf frequency, will be called the equilibrium circle. The azimuth on this circle corresponding to zero rf voltage will be called the equilibrium point, because of an ion whose instantaneous circle coincides with the equilibrium circle

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*"Cyclotron" means the original type, not the fm cyclotron

†One must think here of azimuth as measured from some reference point on the rf cycle. While both ion and reference point are rotating constantly, their relative positions will change only if the particle frequency and the rf frequency are different.

(meaning that the ion has the right frequency) and which always crosses the accelerating gap when the voltage is zero will never lose or gain energy, or change its radius. Finally, it must be remarked that the equilibrium is stable in our sense—an ion displaced from the equilibrium point in azimuth or in energy will be forced back into step by the accelerating voltage, and so will oscillate about the point.

We have shown that an ion can be held in a completely stable orbit, but how can its energy be increased beyond the point described above? This can be done, with some expense to the azimuthal stability, by changing the field and frequency slowly to correspond to a new equilibrium circle of higher energy. If this is done very slowly compared to the time necessary for a round trip of the instantaneous circle around the equilibrium circle, not much harm is done; on the other hand, if the changes are made so fast that the average voltage gain required of the ions per revolution exceeds the available accelerating voltage, then all the ions will surely be lost. For intermediate rates of change, our definition of the equilibrium point must be changed a little, for now the ion which will never oscillate is the one which is just enough ahead of the azimuth corresponding to zero voltage to pick up the correct average voltage gain per turn. The ion that crosses the gap at zero voltage will find after a few turns that its instantaneous circle is too far in, and so will start oscillating about the equilibrium circle.

It is worthwhile noting that the role of the oscillator has now changed completely. In the cyclotron it was a powerful beast used to get the ions through the acceleration period as fast as possible; here it serves to keep the ions in sychronism, any net energy gain coming about as the oscillator tries to compensate for changing equilibrium conditions. This switch of function has an important practical result in that the oscillator may be relatively small, even for a bevatron, since the necessary dee voltage depends only on how fast the field and frequency are changed.

We have seen that it is possible in principle to construct a machine which can control the ions completely at all times during the acceleration period. Before going into detail of actual operation, it might be well to get an idea of the physical appearance of a workable machine, and to look at typical design figures. Figure 1 shows the essential outlines of the accelerator under construction at Berkeley. The highest magnetic field which can be maintained over a sizeable air gap is about 16,000 gauss. For a final proton energy of 6 Bev, this fixes the radius of the machine at about 50 feet. Since it is more or less out of the question to build such a large magnet on the lines of the cyclotron magnet, and indeed, not particularly desirable, the magnet will be ring-shaped, the accelerating chamber taking the form of a big doughnut running through the air gap of the magnet. With this geometry, the field and oscillator frequency must change in such a way that the equilibrium circle remains in the center of the chamber at all times. This means that both must increase during acceleration, the magnetic field linearly with time for practical reasons, and the frequency following along according to the law connecting the energy and velocity of the ions. The magnet is sufficiently large that the electrical equipment feeding it represents a large part of the total cost, with the result that the acceleration period must last for a second or two to keep down power and voltage requirements. This is the longest "duty cycle" of any accelerator attempted to date, and will make the average ion current quite small (10^{-10} to 10^{-9} amps) under the very best conditions. The average energy gain per turn would be about 2000 volts for a 6 Bev machine and about 500 volts for a 1 Bev machine using a slower rate of change of magnetic field, so that accelerating voltages of 4000 volts and 1000 volts respectively would be ample—which is to say that the decrease in the stable range of azimuths and frequency differences caused by the steady change of equilibrium conditions (as mentioned in the last section) would only be by a factor of two or so.

In order to keep the frequency range of the oscillator during acceleration within reason, and to minimize the loss of yield due to ions being scattered in the residual gas (this will be discussed in detail later) it is necessary to introduce the ions into the chamber at as high an energy as possible. At present it is planned to pre-accelerate the ions in either a cyclotron or a Van de Graaff generator to an energy between 4 and 10 Mev before injecting them into the bevatron. During the acceleration, then, the magnetic field would vary from about 200 gauss to its maximum, and the oscillator frequency would range from .25 to 2.5 megacycles/sec. The accelerating electrode, of course, will not be the half pillbox used in the cyclotron, but will probably be localized at one point along the circumference of the

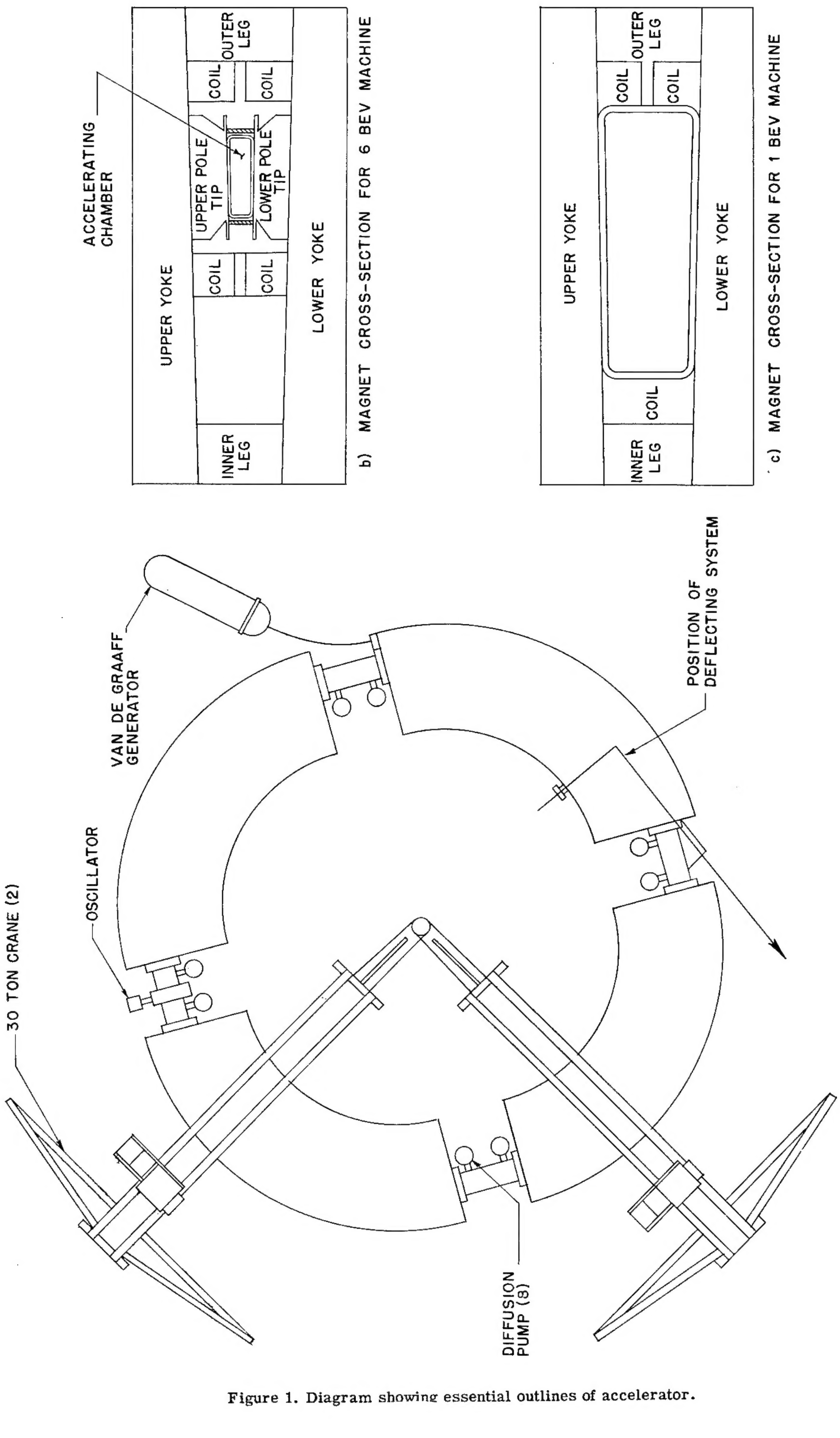

Figure 1. Diagram showing essential outlines of accelerator.

equilibrium orbit, and will resemble a cavity oscillator or a drift tube. A final point, influencing only the appearance of the bevatron, is that four straight sections will be inserted between the four quadrants of the magnet, to give a field-free space in which to introduce the oscillator, injecting system, pumps, and so on. The rest of the accelerating chamber will be pretty well enclosed by magnet iron and coils. The straight sections have, for the most part, negligible influence on the ion orbits while their inclusion is almost a practical necessity. The table summarizes the proposed figures for the two machines.

Finally, a few words about the orders of magnitude involved in the three modes of oscillation of the ions about the stable orbit. The magnetic restoring forces are quite strong, so that the vertical and lateral oscillations run through a cycle in one or two revolutions of the ions. The azimuthal, or synchrotron-type, oscillations are generated by an exchange of energy with the oscillator in many small steps, so that a hundred or more encounters with the oscillator are necessary to carry through one cycle of the oscillation. That the azimuthal oscillations proceed so slowly is the basis for our picture of the ion paths during acceleration. Instead of thinking of the ions as spiralling in and out, we can say that the ions are always moving on their instantaneous circles and that the instantaneous circles are slowly expanding and contracting. At the same time, and superimposed on this picture, the vertical and lateral oscillations are taking place about the instantaneous circles. These fast oscillations may have any amplitude compatible with the size of the accelerating chamber and are determined only by the way in which the ions were started at injection and by the mishaps they may have encountered subsequently. The azimuthal amplitude, on the other hand, will be limited to the stable range determined by the rate at which the field is increasing. For an acdelerating voltage of about twice the average necessary energy gain per turn, this range is about 180°. The corresponding motion of the instantaneous circle covers about 2 feet in the 6 Bev machine, compared to a chamber width of 4 feet; in the 1 Bev machine, where the field rises to about 5000 gauss in 1 second, the amplitude is less than 1 foot, neglibible compared to the 14-foot width of the chamber.

THE OPERATING CYCLE

We are now in a position to follow an ion from injection to the end of its acceleration. This period can be divided into two parts, the injection and the acceleration, of which the first is the most critical, at least for our ideal machine.

Injection

The point has not been stressed but all three types of oscillation which arise are, in fact, quite "pure"—that is, they can be represented as simple sinusoidal oscillations with frequencies depending on the energy of the ion, the shape of the field, the accelerating voltage, and the rate of change of the field. It follows immediately that it is not safe simply to turn things on and start injecting ions, for, since the motion is periodic, it is certain that an ion which leaves the injector by the front door will sooner or later arrive at the back door wanting to repeat its motion, only to run into the injector structure and die. There is a small decrease in amplitude of oscillation during this time which is important during the acceleration period, but it is too slow to be of help in the beginning of the cycle. Instead it is necessary to introduce a non-periodic element into the operation to prevent losing a great number of ions, and it seems easiest to do this as follows: Suppose that the oscillator is not turned on immediately, so that initially the ions are subject only to the changing magnetic field. Let the ions enter the chamber continuously (say from the center of the outside wall—then there will be no vertical oscillations about the center plane). There will come a moment when the instantaneous circle corresponding to the energy of the injected ions will reach the outside wall, moving inward. From this time on, the injected ions will ride inward with the instantaneous circle, the first ones following it exactly and later ones oscillating about it because they will start at a finite distance from it and will be pulled toward it by the magnetic force. Because the instantaneous circle is not oscillating but is always moving inward, the

BEVATRON

BRIEF SPECIFICATIONS ON WHICH DESIGN IS NOW PROCEEDING

		Initial stage	Later stage
Aperture inside vacuum	ft	4 x 14	1 x 4
Radius to center of orbit	ft	46	50
Length of straight sections	ft	15	15
Maximum proton energy	Bev	1.36	6.44
Maximum magnetic field	gauss	5,000	16,000
Magnet gap	in	54	14
Injection energy	Mev	4	4
Minimum radio frequency	mc/sec	0.27	0.24
Maximum radio frequency	mc/sec	2.58	2.61
Injection field	gauss	216	190
Acceleration time	sec	1.4 1.6	1.4
Initial rate of rise of field	kG/sec	4.5	13.2
Initial energy gain per turn	ev	600	2100
Repetition rate - pulses per min		15	15
Quantity of iron in magnet	tons	8700	10,200
Stored energy in magnet - megajoules		80 megajoules	
Turns on magnet		88	
Peak magnet current		8,000 amps	
Peak ampere turns		704,000	
Initial voltage on magnet		18,750 volts	
Voltage at maximum current		12,500 volts	
Peak instantaneous power		100 megawatts	
Motor power rating		5800 hp	
Conductor weight		350 tons	
Conductor cross section		1.31 sq in	
Total length of conductor		140,500 ft	
Number of conductors in parallel		2	

ions will be carried clear of the injector even though the amplitude of the lateral oscillations about the instantaneous circle does not decrease. Figure 2 shows the paths of ions entering the chamber at different times during this period; it can be seen that ions entering after the instantaneous circle passes the center of the chamber will strike the inner wall during their first lateral oscillation.

Now if the oscillator is turned on just when the instantaneous circle reaches the center of the chamber it will see a chamber full of ions, all of the same energy and therefore all traveling on the same instantaneous circle, but with all possible lateral amplitudes from zero to the half width of the chamber, depending on when the particular ions entered the chamber. From this distribution it will select those in the stable azimuthal range and accelerate them, while the others will be lost. Even some in the stable range will be lost because radial oscillations of the instantaneous circle may cause them to hit the chamber walls if their lateral amplitudes are large. In the 6 Bev machine, where the radial motion covers about half the chamber, the additional loss is about 20 or 30 per cent if the accelerating voltage is twice the necessary voltage gain per turn. In the 1 Bev machine, where the radial motion is negligible, there is no additional loss.

We can see the conflicting factors governing the number of ions caught. To get as many ions into the chamber as possible before the oscillator is started, the field should rise very slowly, so that the instantaneous circle will move in slowly. But this means less clearance for the ions at the injector, for we have depended on the motion of the instantaneous circle to get them past. At present, the latter

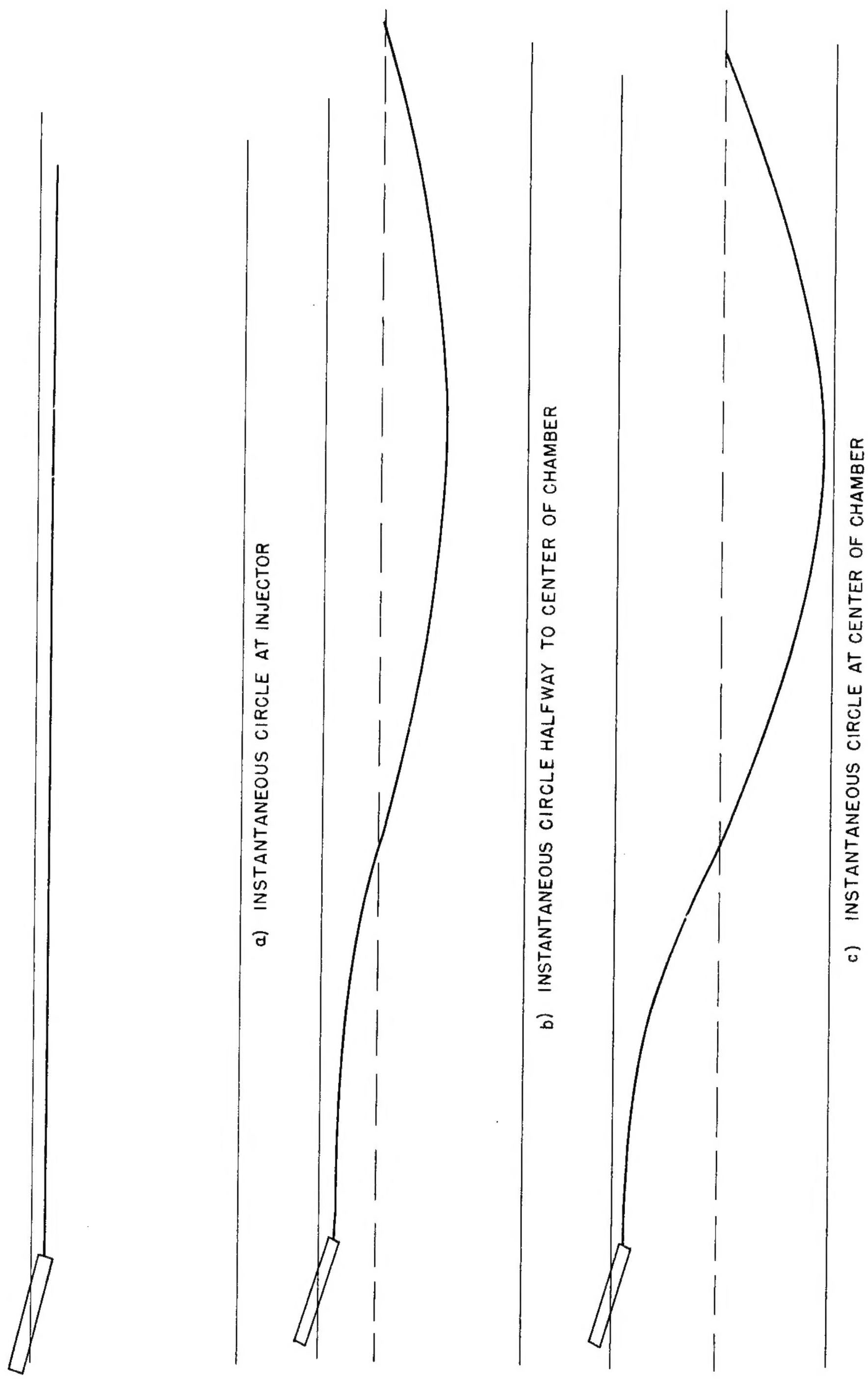

Figure 2. Ion paths during injection period.

seems the more serious point, for the circle moves only .2 inch per revolution in the 1 Bev machine, and so all efforts are toward making the circle move in as fast as possible, by increasing the rate at which the field falls off with radius and by making the rate of change of field as large as possible (see also the section on gas scattering).

After the oscillator is started there is another conflict. If we make the accelerating voltage very large compared to the average energy gain per turn, we will achieve the same result as by making the average energy gain per turn very small — we will approach the case of constant field and frequency in which the stable azimuthal range will cover the whole circumference of the machine. But at the same time, increasing the dee voltage means that, as an ion oscillates in azimuth, it will be suffering much larger gains and losses of energy, and the instantaneous circle will swing farther on each side of the equilibrium circle thus causing more ions to strike the walls. In the 6 Bev machine, an accelerating voltage of twice the average energy gain per turn is about the optimum; in the 1 Bev machine the yield could be improved considerably by using a higher voltage.

Behavior of Ions During Acceleration

From the above we can picture the paths of the ions during acceleration. If we forget about the fast lateral oscillations, which are simply superimposed on the basic motion, then at the moment at which the oscillator is turned on, all the ions are spread evenly along the equilibrium circle. About 180° of this circle is accepted by the oscillator as previously mentioned. The ion at the equilibrium point will stay in place, increasing its frequency exactly in step with the oscillator. The ions behind and ahead of it will begin their azimuthal oscillations about the equilibrium point, which an accompanying in-and-out motion of their instantaneous circles. This means that the original line of ions will begin to rotate about the equilibrium point. We have said that the oscillations are almost purely sinusoidal, which would imply that this line rotates as a straight line, since all ions would have the same frequency of oscillation. Actually, the frequency depends somewhat on the amplitude, so that the line will soon bend, and eventually the ions will be pretty evenly smeared out over an oval region, with the equilibrium point approximately at the center. These three stages are illustrated in Figure 3, and it must be imagined that the lateral oscillations are superimposed on these diagrams to show the complete motion.

This "bunch" will move through the machine with increasing velocity as the acceleration progresses without essentially changing its shape.

It is necessary at this point to admit that our definition of stable orbits was not quite complete. In addition to requiring the existence of a path such that an ion displaced from it will be forced back toward it, we must also stipulate that the oscillations generated in this way shall not increase their amplitudes in time. If we represent the three types of oscillation by three model springs whose stiffness and inertia are related to the energy of the particle, the shape of the field, and so on, it is apparent that if these quantities were constant in time, the springs would go on vibrating with their initial amplitudes forever, there being no friction to damp them down, or driving forces to build them up. But these quantities are not constant, and so we have vibrating springs whose stiffness and inertia are changing slowly in time, and we must be sure that these changes do not act in such a way that the oscillation amplitudes are increased by them. A calculation shows that the amplitudes do not increase; in fact, the amplitudes decrease in time, though rather too slowly to be of assistance during the injection period. However, by the end of the long acceleration period, the bunch and the superimposed oscillations will have been reduced in size to a region of several inches diameter around the equilibrium point. This effect has a practical consequence in that the tolerance on keeping the equilibrium circle in the center of the accelerating chamber can be relaxed considerably toward the end of the acceleration.

Removal of the Beam

What to do with the ions after they reach peak energy is not a question of primary importance at this time. It would be a simple matter to bring the beam to the wall of the chamber by upsetting the

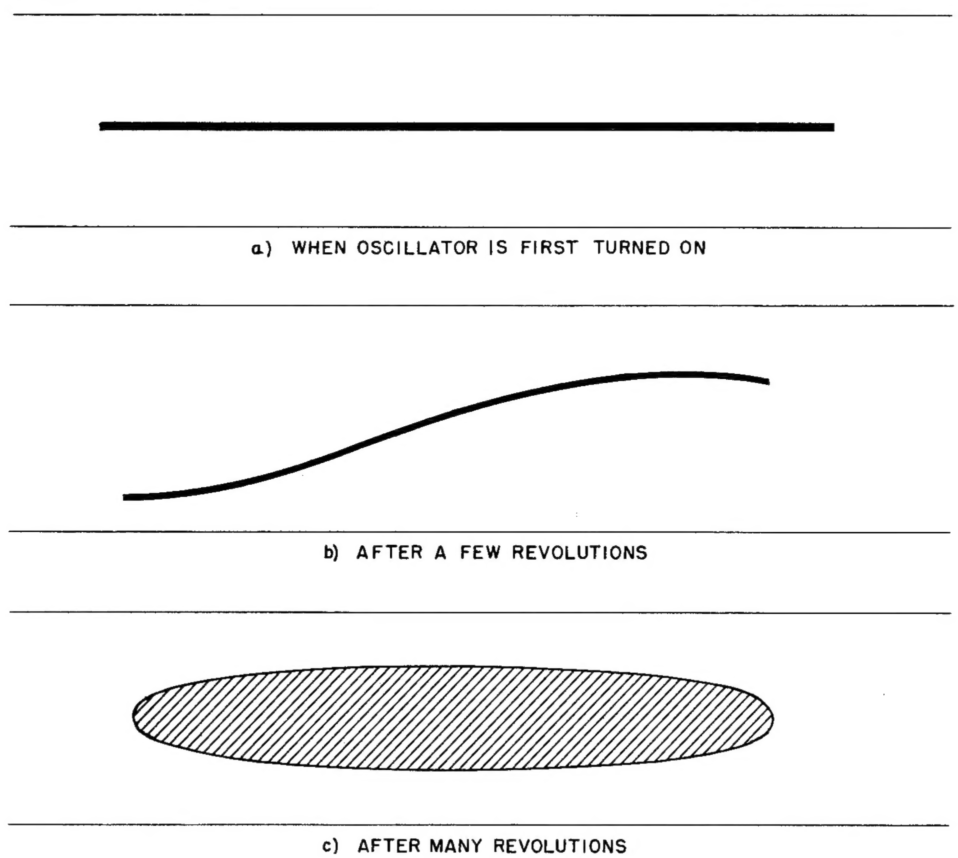

Figure 3. Ion bunch during acceleration (lateral oscillations are not shown).

field-frequency relation, from where it could probably be brought out of the chamber by electric or magnetic deflection. Even without this, experience with the 184-inch fm cyclotron shows that many interesting things can be done with probes and other devices which could be inserted in the straight sections of the chamber.

Practical Considerations

Since it is not possible to build the perfect machine which we have been discussing up to this point, it is now essential to inquire how serious the effects of imperfections in the actual machine will be. Mathematical methods are badly limited in such a situation to using simplifying assumptions, but can at least predict the effects of certain types of imperfections and thus provide a partial guide in design. We shall consider a number of different types of deviations from the ideal case, and attack them in such a way as to get results in the form of tolerances on the various quantities involved.

Gas Scattering

The effect of scattering of the ions by the residual gas in the chamber demands close attention, for the ions must travel a distance equal to several times the circumference of the earth without deviating more than a few degrees from the equilibrium orbit. The average deflection of an ion on striking a gas molecule is very small, but there is ample time for them to add up during the long acceleration period.

First, let us note how various factors must affect the scattering loss. Of course, high gas pressure and high atomic number for the residual gas are bad. A short acceleration period would leave less time for scattering. Strong magnetic focussing would minimize the effect of the individual scatterings, and a big chamber would extend the time needed to build up an amplitude of oscillation which would mean catastrophe. Finally, since scattering probabilities decrease rapidly with increasing ion energy, a high injection energy should be very helpful.

The functional form in which these factors appear can be seen by considering a simple case. Suppose that every time an ion strikes a gas atom it is lost to the beam and suppose that the scattering probability did not decrease with increasing energy. Then the final intensity would be a decreasing exponential function of the distance gone, just as in the case of neutrons or gamma rays being absorbed in matter. The "absorption coefficient" would contain factors mentioned in the preceding paragraph. Actually, it takes many collisions to cause a catastrophe, and the scattering probability decreases rapidly in time. A calculation shows, however, that these effects do not destroy the exponential form, but the decreased scattering probability causes the scattering above 40 Mev or so to have a negligible effect, compared to that between 4 and 40 Mev.

The calculation shows that for a pressure of less than 10^{-5} mm, the scattering should not be serious in either machine. However, the figures are based on the geometrical dimensions of the chamber, so that if the effective dimensions are smaller because of an error in oscillator frequency or asymmetries in the magnetic field, the yield could be much improved by further reducing the pressure, or changing the other variables. Because of the exponential form of the dependence, the scattering loss is very sensitive to changes; for example, if only one ion in a million were getting through at an injection energy of 4 Mev, then almost all would get through if the injection energy could somehow be raised to 30 Mev or so.

It is important to remark that the scattering is the only imperfection which influences the yield more strongly than in simple proportion to the amount of imperfection. For this reason it should probably be a major influence in deciding on final design figures. Other tolerances may be sacrificed more easily if there are good reasons of a mechanical or structural nature for doing so.

Missing the Injector

The question of how many ions will succeed in avoiding the injector and the chamber walls during the injection period is probably next in order of importance. For the ion which leaves the injector in a direction parallel to the wall just as the instantaneous circle is at the injector, the answer is easy. In

the 6 Bev machine the instantaneous circle moves approximately .8 inch, and in the 1 Bev machine about .2 inch, per revolution, so that these ions will strike the injector the second time around if the injector structure protrudes more than this amount beyond the point at which the ions actually enter. For ions entering at other times and in other directions, however, it becomes necessary to follow them for several revolutions in order to discover whether or not they will run into something. We shall now discuss the behavior of the ions during the first few revolutions in more detail.

One effect should be very helpful, namely, that the frequency of lateral oscillation is close to, but somewhat more than half the rotation frequency, so that the oscillations do not bring the ions to a maximum displacement from the instantaneous circle while in the vicinity of the injector for five or ten revolutions. Hence, the actual clearances are five or ten times larger than the distance moved by the instantaneous circle per revolution. In the first betatron, this precession of the lateral oscillations with respect to the rotational motion was given credit for all the electrons which managed to miss the injector; today, however, it seems that the capture mechanism in the betatron is somewhat different from what was intended so that the success of the betatron does not imply that this precession is the best insurance against losing ions. If the injected beam enters as a fine pencil, so that the injector structure may be limited in height, then the effect of precession may be extended considerably by pointing the beam slightly up or down. The vertical oscillations will then also precess, and at a rate different from that of the lateral oscillations, so that many more turns take place before the ion again approaches the injector.* If, on the other hand, the ions must be sprayed in over the entire height of the chamber, then only the radial oscillations can be of assistance.

From the above description, it seems almost certain that the ions which leave the injector in a direction parallel to the wall (and thus at their maximum distance from the instantaneous circle) will clear the injector with perhaps an inch to spare. Therefore if the injected beam is sufficiently narrow in angular spread, all ions can be made to miss the injector by bringing them in parallel to the wall. If the beam has a sizeable spread, then we must ask what range of initial directions is permissible. Estimates based on following the lateral oscillations for a number of revolutions indicate an angular range of the order of a degree.

The permissible vertical angular range in which ions injected at the center plane may enter the chamber and not strike the top or bottom can be estimated more precisely, since the angle of crossing the center plane determines the amplitude of the sinusoidal oscillation. For a 1-foot aperture, the angle is approximately one degree; for a 4-foot aperture, approximately four degrees.

Thus there is a finite range of directions from the injector in which ions may enter and be accelerated. It may be possible to improve on this solid angle by imitating the injection technique which seems to be behind the success of the betatron, but details of this method have not yet been fully explored.

Frequency Tracking

The fact that the magnet is ring-shaped creates a worry which did not exist in the case of the fm cyclotron, in which the radial clearance is unlimited. The oscillator frequency must have the proper relation to the magnetic field to keep the equilibrium orbit in the center of the chamber—a discrepancy between field and frequency would mean that the circle defined jointly by them will move from the center of the chamber, thus risking that some of the ions oscillating about it will be driven into the wall.

In the initial stages of acceleration, the width of the chamber will be evenly filled with ions oscillating in the radial direction. A shift of the equilibrium circle will then cause a proportionate number of ions to strike the wall. If the discrepancy is so great that the equilibrium circle is not even in the chamber, then all will be lost. At the other extreme of the acceleration period, when the oscillations

* The exact number of revolutions depends on the initial distance from the instantaneous circle, the direction of injection, and the strength of the focussing forces.

of all the ions will be dampened to having very small amplitudes about the equilibrium point, only a drastic shift of the circle would do noticeable harm.

Calculation of the frequency tolerance is simple if the energy of the ions is high, for then they travel at practically constant speed, so that the per cent shifts in radius and frequency are equal. Thus the final frequency tolerance will be equal to the half-width of the chamber divided by the radius; for the 6 Bev machine this is ± 4%, and for the 1 Bev machine, ± 14%. At low energies the tolerance also depends on the shape of the field. This lowers the tolerance for 100% loss of ions to ± 3% and ± 10%, respectively. Since the distribution of ions in the bunch is more or less uniform, tolerances for smaller losses will be in proportion. For example, the loss will be under 10% if the frequency is within ± .3% and ± 1% of the correct values, respectively.

Asymmetries in the Field

Asymmetries in the magnetic field can, to a first approximation, be supposed to consist of two separate parts: first, a variation of the vertical component of the field along the circumference of the equilibrium circle, and second, a warping of the surface on which the radial component of the field is zero, a surface which coincides in the ideal case with the geometrical center plane of the magnet gap. Although Maxwell's equations relate the vertical and radial components, these two particular types of distortion can exist independently, having only secondary influence on each other. The angular variation of field affects the lateral motion of the ions; the warping of the center plane, their vertical motion. The situation is now changed in that the ion feels, in addition to the normal magnetic restoring force, a force which has a basic frequency equal to the frequency of rotation. Thus we can say that our representative springs are subject to driving forces of possibly complicated form.

It is most convenient to resolve such a force into its Fourier components, using the rotation frequency as the fundamental. We can then speak of 1st harmonic, 2nd harmonic, etc., deviations and discuss their effects more or less independently. There is an immediate and general consequence to this way of looking at the problem--since the "natural" (without driving force) vertical and horizontal oscillation frequencies are always less than the rotation frequency, the difference between the natural frequency and that of the n^{th} harmonic driving force will increase with n. Therefore we can call on our knowledge of the behavior of resonating systems when driven by a force off resonance—the oscillation amplitude built up by the high harmonic components will be very small. For example, this bears out what one would suspect about the effects on the orbits of the gaps between magnet laminations—they are negligible.

Accordingly we can forget about local bumps in the field, even though they may be quite large, and worry only about the first harmonic variation, which is to say, about the average variations between a given point in the field and the point diametrically opposed to it. For the lateral oscillations, this amounts to a shift in the center of the instantaneous circle, and we must require that this shift be small enough that all points of the equilibrium circle lie within the chamber. For the vertical motion, the first harmonic variation represents a tilt of the center plane, and we must require that ions do not strike the top or bottom as they run uphill and down. The tolerances which follow on this basis for the two machines are:

	6 Bev	1 Bev
% 1^{st} harmonic variation	±.4%	±1%
Maximum 1^{st} harmonic Displacement of median Surface from geometric center	±2 inches	±8 inches

At injection time, the tolerances on the angular variation of the field depend also on the orientation of the displacement of the equilibrium circle with respect to the injector, and in the best case they may

be slackened by as much as 100%. Our estimate of the effective solid angle of injected ions, which was based on calculations of ion paths as the ions leave the injector, would be seriously influenced by an angular variation in magnetic field. This may be dangerous, but it suggests that it may be possible to assist the ions during the injection period by a proper distortion of the field.

Shape of Field

In picking a particular rate of decrease of field with radius, we shall try to assist the instantaneous circle in moving away from the injector as much as possible, and at the same time try to avoid the possibility of a resonant increase in vertical and lateral amplitudes of oscillation due to interactions between the vertical, lateral, and rotational motions (the azimuthal oscillations are not involved in questions of resonance because their period is so much longer than that of the other types). The shape of the field is of importance in deciding if resonances will occur, for it alone determines the relation of the three frequencies.

Interactions between the vertical and lateral oscillations will always exist in an actual magnet—the lateral force on an ion will always depend a little on its vertical displacement from the center plane. The mechanical analogy is gained by stretching a weak coupling spring between the two representative springs, so that the restoring force on each will depend slightly on what the other is doing. An interaction of oscillation with the rotational motion will occur if the field is not uniform along the circumference of the orbits, or even because the acceleration of the ions is not continuous but consists of small kicks of a frequency equal to the rotational frequency. These interactions act as a driving force on the oscillations, as developed in the last section.

These couplings have, in general, no consequences beyond those treated in the previous section. However, if there is a commensurability between the vertical and lateral frequencies, oscillation energy may pass steadily from one mode to the other, with a corresponding increase in amplitude which may be fatal, as it is in the 184-inch cyclotron. For the coupling with the rotational motion, it is clear that if the oscillation frequencies were equal to the rotation frequency, the amplitude would build up steadily under the action of the driving force, but actually there can be danger even if the frequencies are not equal. This latter danger comes about because the oscillations are not exactly pure sine waves, so that higher Fourier components of the motion will have the same frequency as one of the harmonic components of the driving force if the fundamental frequencies are commensurable.

Choosing a field shape then amounts to finding a range of shapes, as steep as possible, and free from commensurability relations of low order. Only frequency ratios of 1:1 and 1:2 should be really dangerous, for the sharpness of the resonant region decreases so rapidly with increasing order that an ion will run out of the dangerous region of the field as soon as the amplitude begins to increase.

In calculating the frequencies of oscillation from the field shapes, it must be remembered that the field-free sections of the chamber serve to change the frequency, for they periodically introduce a change in phase of the sinusoidal oscillations. Taking this into account, one finds a suitable range free of small order commensurabilities with a field strength proportional to $r^{-.72}$ in the center of the range, and so this shape was chosen. It should be added that a change in the length of the straight sections affects the frequencies, so that the shape of the field would have to be reconsidered. In particular, an increase in length would seem to make it wise to use a flatter field.

Relative Dimensions of the Chamber

There is also a question as to what is the proper relation between the height and width of the accelerating chamber. In this matter, we must be guided by the relative ease of obtaining the various tolerances discussed, and by the effect of scattering loss. The tolerances probably make no particular demands on the proportions of the chamber; it is the purpose of this section to point out the role of the scattering loss.

According to the section on scattering losses, the negative exponent determining the number of ions which reach the end of the acceleration must vary inversely with dimensions of the chamber and also inversely with the strength of the focussing forces. Since ions may be lost either to the walls or the top and bottom, the exponent must vary as the sum of factors representing these two combinations of variables for the two motions involved. Accordingly, some idea of the best dimensions of the chamber, say for a given volume, can be obtained by minimizing this expression. For the field shape chosen, the best ratio of height to width is about 1:2 if the motion of the instantaneous circle is not great enough to decrease appreciably the effective width of the chamber for lateral oscillation amplitudes. In the 6 Bev machine, where this motion is not negligible, the best ratio is about 1:4, which is the ratio actually chosen. It must be stressed that these figures are based on a given volume of chamber; if the height is fixed instead, then of course the chamber should be as wide as possible.

An additional argument for width is that this would increase the fraction of the chamber width which is in useful field; i.e., in a field region where the shape is the desired one. Thus there seems to be no strong argument for choosing proportions different from those on which the designs are based at present.

Shielding

It is difficult to predict the type and amount of shielding needed to make the bevatron safe, for one of the primary purposes of building a large machine is to investigate the nature of the processes by which a high energy particle is slowed down. The only hint as to how these particles will behave comes out of cosmic ray experiments. According to the best measurements to date, protons in the energy range of 1 to 10 Bev produce many mesotrons when passing through matter and dissipate their energy rapidly in this way, the number of mesotron-producing protons decreasing by a factor 2 in about 100 to 200 gm/cm^2 of matter. It is supposed that they lose their ability to create mesotrons rapidly when their kinetic energy drops below 1 Bev.

If we except these statements, then a few feet of a heavy material should be sufficient to bring the particles (which, incidentally, may have changed to neutrons in the process) out of the high energy range. From that point on the shielding problem would be similar to that of the 184-inch cyclotron, except that the intensity will be 100 times smaller. Because of the low intensity, the rapid dissipation of energy, and a decided preference for the forward direction in nuclear collisions, personnel should be safe if the beam is pointed into the hill and five feet or so of shielding is set up on the opposite side, in front of laboratories and offices. It looks, in fact, as though the pre-accelerator used as an injector might be just as dangerous as the bevatron itself. However, we have practically no knowledge at this point of the behavior of very high energy nucleons, so that it would be wise to keep in mind the safety problem when laying out the laboratory, so that much more shielding may be added, or the control rooms moved, if it proves necessary.

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